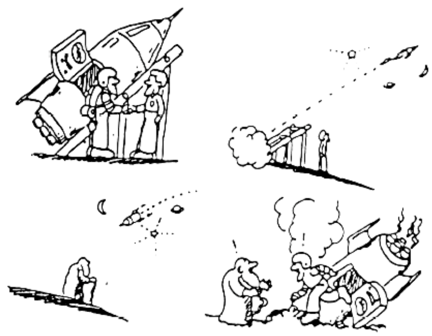
Name ______________________________ Date ____________

CONCEPTUAL **Physical Science** PRACTICE SHEET

Chapter 30: Relativity and the Universe
Relativistic Time Dilation



This is about identical twins, one an astronaut who takes a high-speed round-trip journey while the other twin stays home on earth. The traveling twin returns younger than the stay-at-home twin. How much younger depends on the relative speeds involved. If the traveler maintains a speed 0.5c for 1 year (according to clocks aboard the spaceship), 1.15 years elapse on earth. For a speed of 0.87c for a year, 2 years elapse on earth. At 0.995c, 10 earth years pass in one spaceship year; the traveling twin ages a single year while the stay-at-home twin ages 10 years.

This exercise will show that from the frames of reference of both twins, the earthbound twin ages more.

Case 1: No Motion First, consider a spaceship hovering at rest relative to a distant planet (Figure 1). Suppose the ship sends regularly-spaced brief flashes of light to the planet. The light flashes encounter a receiver on the planet a slight time later at speed *c*.

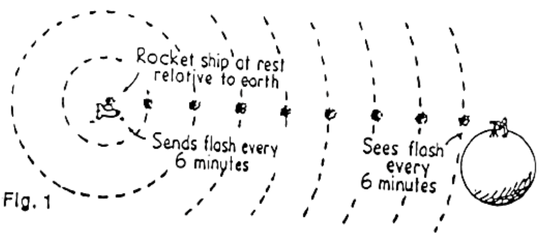


Fig. 1

Since there is no relative motion between sender and receiver, successive flashes are received as frequently as they are sent. We'll suppose a flash is sent from the ship every 6 minutes; after slight delay, the receiver sees a flash every 6 minutes. Nothing unusual, because no motion is involved.

Case 2: Motion For motion the situation is quite different. Although the speed of the flashes are *c*, regardless of motion, how *frequently* the flashes are seen very much depends on relative motion. When the ship approaches the receiver, the receiver sees the flashes more frequently. This makes sense because each succeeding flash has less distance to travel as the ship gets closer to the receiver. Flashes are "crowded together" and are seen more frequently. Flashes sent at 6-min intervals are seen as less than 6 min apart. We'll suppose the ship is traveling fast enough for the flashes to be seen twice as frequently, at intervals of 3 min (Figure 2). This is the Doppler effect (Chapter 10) for light.

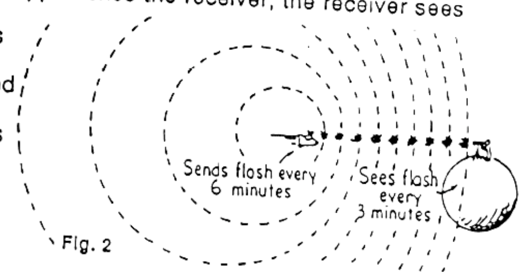


Fig. 2

Motion away from the receiver stretches the flashes apart and they are seen less frequently. If the ship recedes from the receiver at the same speed and still emits flashes at 6-min intervals, these flashes are seen stretched out to 12-min intervals by the receiver. Put another way, they will be seen half as frequently, that is, one flash each 12-

min interval (Figure 3). This makes sense because each succeeding flash has a longer distance to travel as the ship gets farther away from the receiver.

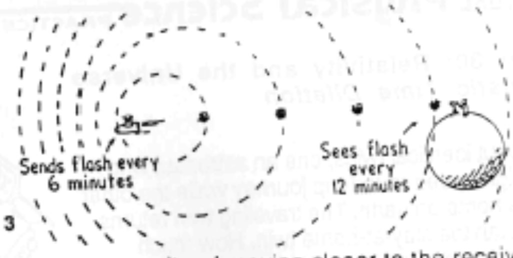


Fig. 3

Note the effect of moving away is just the opposite of moving closer to the receiver. Flashes are received twice as frequently when the spaceship is approaching (6-min flash intervals are seen every 3 min), and are received half as frequently when receding (6-min flash intervals are seen every 12 min).

The light flashes make up a light clock. Any reliable clock would show that in the receiver's frame of reference, events that take 6 min in the spaceship are seen to take 12 min when the spaceship recedes and only 3 min when the ship is approaching.

1. If the spaceship travels for 1 h and emits a flash every 6 min, how many flashes will be emitted? ____________

2. The ship sends equally spaced 6-min flashes while approaching the receiver at constant speed. Will these flashes be equally spaced when they encounter the receiver? ____________ How about if the ship is accelerating when sending flashes? ____________

3. If the receiver sees these flashes at 3-min intervals, how much time will occur between the first and the last flash (in the frame of reference of the receiver)?

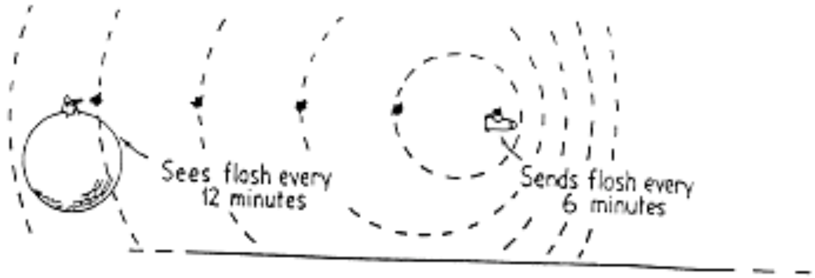
____

Case 3 The Twins Let's apply all this to the twins. Suppose the traveling twin leaves the earthbound twin at the same high speed for 1 hour and then quickly turns around and returns in 1 hour. The traveling twin takes a round trip of 2 hours, according to all clocks aboard the spaceship. The time for the round trip will be something else from the earth frame of reference!

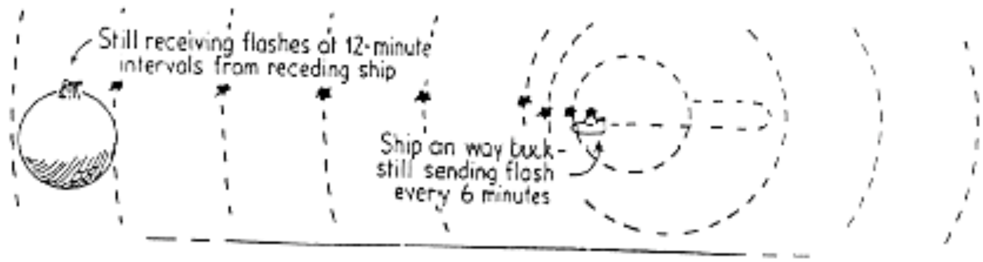
In Figure 4a we see the ship receding from earth, emitting a flash each 6 min. Due to motion, flashes are received on earth every 12 min. During the hour of going away from earth, a total of ten flashes are emitted. If the ship departs from the earth at noon, clocks aboard the ship read 1 PM when the tenth flash is emitted. What time will it be on earth when this tenth flash reaches the earth? The answer is 2 PM. Why? Because the time it takes the earth to receive 10 flashes at 12-min intervals is 10 x (12 min), or 120 min (= 2 hours).

Suppose the spaceship turns around suddenly in a negligibly short time and returns at the same high speed. During the hour of return it emits another ten flashes at 6-min intervals. These flashes are received every 3 min on earth, so all ten flashes come in 30 min. A clock on earth will read 2:30 PM when the spaceship completes its 2-hour trip. This means the earthbound twin has aged 1/2 hour more than the twin aboard the spaceship!

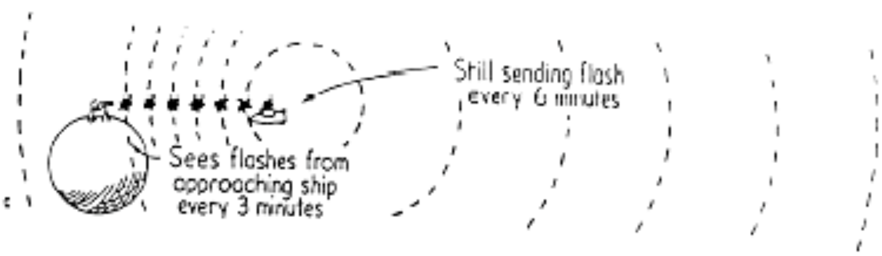
Fig. 4



a

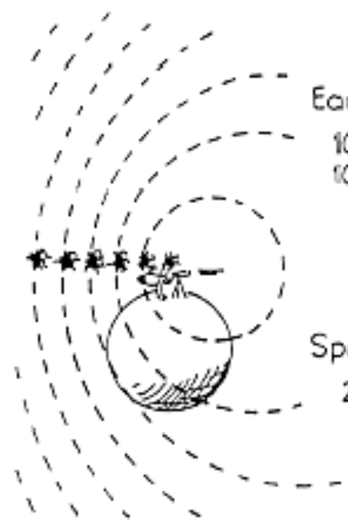


b



c

Complete Figure 5, which summarizes Case 3, by filling in the blanks.



Earth frame of reference:

10 flashes @ 12 min = ____ minutes
10 flashes @ 3 min = ____ minutes
minutes

____ hours

Spaceship frame of reference:

20 flashes @ 6 min = ____ minutes

____ hours

Fig. 5

Case 4 Sending and Receiving Twins Interchanged Let's switch sender and receiver and see if the result is the same from either frame of reference. Flashes are emitted from the earth at regularly spaced 6-min intervals in earth time, but are seen from the frame of reference of the receding spaceship at 12-min intervals (Figure 6a). This means that a total of five flashes are seen by the spaceship during the hour of receding from earth. During the spaceship's hour of approaching, the light flashes are seen at 3-min intervals (Figure 6b), so 20 flashes will be seen.

So the spaceship receives a total of 25 flashes during its 2-hour trip. According to clocks on the earth, however, the time it takes to emit the 25 flashes at 6-min intervals is 25 x (6 min), or 150 min (= 2.5 hour).

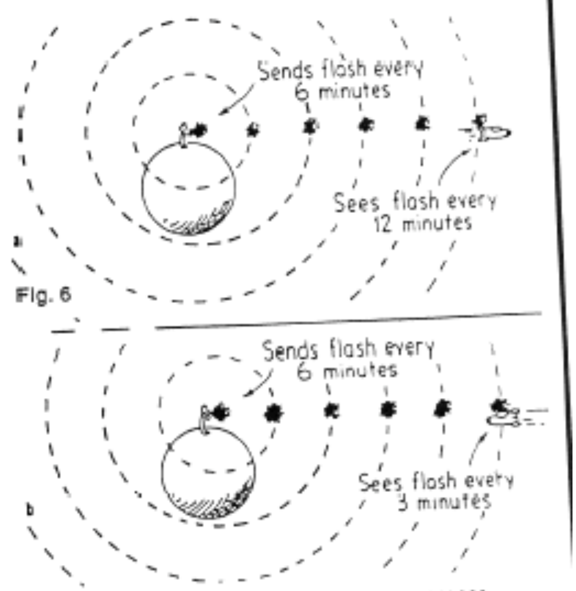
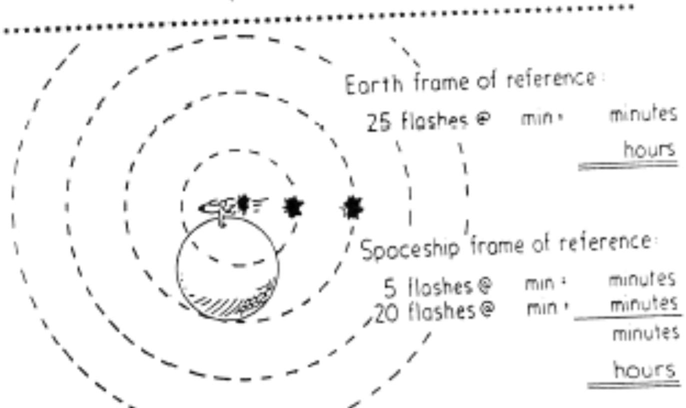


Fig. 6

Complete Figure 7, which summarizes Case 4, by filling in the blanks.



Conclusion So both twins agree on the same results, with no dispute as to who ages more. The key factor is that while the stay-at-home twin remains in a single reference frame, the traveling twin has experienced two different frames of reference, separated by the acceleration of the spaceship in turning around. The spaceship experiences two different realms of time, while the earth experiences a still different but single realm of time. The twins can meet again at the same place in space only at the expense of time.

The twin example is often called the twin "paradox" because of the following reasoning: Since motion is relative, the rocket ship can be regarded at rest and the earth moves, in which case the twin on the rocket ship ages more. Question: Is the situation symmetrical; that is, do both twins occupy the same realms of time? __________ What event separates the _________ realms of time for the traveling twin? ______________ So is this twin-paradox reasoning correct or incorrect? ____________ Briefly, and in ... why? ______________

Name ______________________________ Date ____________

Chapter 30: Relativity and the Universe
Time Dilation and the Twin Trip

This practice sheet recaps the *twin trip* of the previous practice sheets, where a traveling twin takes a 2-hour journey while a stay-at-home brother records the passage of 2 $^1/_2$ hours. Quite remarkable! Times in both frames of reference are marked by flashes of light, sent each 6 minutes from the spaceship, and received on Earth at 12-min intervals for the ship going away, and 3-min intervals for the ship returning. Fill in the clock readings aboard the spaceship when each flash is emitted, and the clock reading on Earth when each flash is received.

SHIP LEAVING EARTH

FLASH	TIME ON SHIP WHEN FLASH SENT	TIME ON EARTH WHEN FLASH SEEN
0	12:00	12:00
1	12:06	
2		
3		
4		
5		
6		
7		
8		
9		
10		

SHIP APPROACHING EARTH

FLASH	TIME ON SHIP WHEN FLASH SEEN	TIME ON EARTH WHEN FLASH SEEN
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

THIS CHECKS: FOR $v = 0.6c$

$$t = \frac{t_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{2\text{ hr}}{\sqrt{1-\left(\frac{0.6c}{c}\right)^2}} = 2.5\text{ hr}$$

15-1 Conceptual PHYSICS Next-Time Question

AN ASTRONAUT AGES 3 YEARS WHEN TRAVELING AT 99% THE SPEED OF LIGHT TO THE STAR PROCYON AND BACK. THE SPACE OFFICIALS TO GREET HER ON HER RETURN AGE

a) LESS THAN 3 YEARS

b) 3 YEARS

c) MORE THAN 3 YEARS

16-1 Conceptual PHYSICS Next-Time Question

A 1-METER LONG SPEAR IS THROWN AT A RELATIVISTIC SPEED THROUGH A PIPE THAT IS 1 METER LONG. BOTH THESE DIMENSIONS ARE MEASURED WHEN EACH IS AT REST. WHEN THE SPEAR PASSES THROUGH THE PIPE, WHICH OF THESE STATEMENTS BEST DESCRIBES WHAT IS OBSERVED?

a) THE SPEAR SHRINKS SO THAT THE PIPE COMPLETELY COVERS IT AT SOME POINT

b) THE PIPE SHRINKS SO THAT THE SPEAR EXTENDS FROM BOTH ENDS AT SOME POINT

c) BOTH SHRINK EQUALLY SO THE PIPE COMPLETELY COVERS IT AT SOME POINT

d) ANY OF THESE, DEPENDING ON THE MOTION OF THE OBSERVER