

PHYSICS SKILL

Dimensional Analysis

Quantities such as length, speed, and area are called dimensional quantities. A measured dimension, quantity has a numerical value that depends upon the system of units used. For example, a given area can be stated as either 1 square meter or 10 000 square centimeters. When making a measurement, the most convenient unit is used.

When substituting values into an equation in physics, you must state the units as well as the numerical values. Including units in your calculations helps you keep units consistent throughout and assures you that your answer will be dimensionally correct. You may also use the units or dimensions of your measurements to check the correctness of your equation.

Example

Dimensionally Incorrect Equation

$$\begin{aligned} \text{velocity} & \stackrel{?}{=} (\text{distance}) (\text{time}) \\ \frac{\text{m}}{\text{s}} & \stackrel{?}{=} \frac{\text{m}}{\text{s}} \\ \frac{\text{m}}{\text{s}} & \neq \text{m} \cdot \text{s} \end{aligned}$$

Note that the final units on the right do not equal those on the left. By inspecting the dimensions, you should be immediately aware that the equation is not correctly written.

Example

Dimensionally Correct Equation

$$\begin{aligned} \text{mass} & \stackrel{?}{=} (\text{density}) (\text{volume}) \\ \text{kg} & \stackrel{?}{=} \frac{\text{kg}}{\text{m}^3} \text{m}^3 \\ \text{kg} & = \text{kg} \end{aligned}$$

Note that some of the units on the right side of the equation cancel out. The final dimensions on both sides are the same. The equation is dimensionally correct.

Problems

Use the method described above to determine if the following equations are correctly written. The proper units for the variables are listed below.

d (distance)-meters

v (velocity)-meters/second

t (time)-seconds

a (acceleration)-meters/(second)²

v_i = initial velocity; v_f = final velocity; v = average velocity

1. $v_f \stackrel{?}{=} v_i t + a$

2. $\bar{v} \stackrel{?}{=} \frac{v_i + v_f}{2}$

3. $d \stackrel{?}{=} \sqrt{v_i t^2 + \frac{1}{2} a t}$

4. $v_f \stackrel{?}{=} v_i^2 + 2 a d^2$

5. $d \stackrel{?}{=} \frac{1}{2} v_i t + a t$

6. If r is in meters
 g is in meters/s²
 x is in kg/meters³
 n is in kg/meter s
 does $v = \frac{2 r^2 g x}{9 a n}$?

PHYSICS

Factor-Label Method for Converting Units

A very useful method of converting one unit to an equivalent unit is called the factor-label method of unit conversion. You may be given the speed of an object as 25 km/h and wish to express it in m/s. To make this conversion, you must change km to m and h to s. In algebra, you learned that if a quantity is multiplied by 1, its value does not change. But 1 is just a quantity divided by its equivalent. Since $1000 \text{ m} = 1 \text{ km}$ and $60 \text{ s} = 1 \text{ min}$ and $60 \text{ min} = 1 \text{ h}$,

$$\frac{1000 \text{ m}}{1 \text{ km}} = 1 \qquad \frac{1 \text{ min}}{60 \text{ s}} = 1 \qquad \frac{1 \text{ h}}{60 \text{ min}} = 1$$

To change 25 km/h to m/s you must multiply by a series of factors so that the units you do not want will cancel out and the units you want will remain.

$$\frac{25 \cancel{\text{ km}}}{1 \cancel{\text{ h}}} \times \frac{1000 \text{ m}}{1 \cancel{\text{ km}}} \times \frac{1 \cancel{\text{ h}}}{60 \cancel{\text{ min}}} \times \frac{1 \cancel{\text{ min}}}{60 \text{ s}} = 6.9 \text{ m/s}$$

To convert 80 milliliters to liters, first choose the factor. Since $1 \text{ L} = 1000 \text{ mL}$,

$$\frac{1 \text{ L}}{1000 \text{ mL}} = 1$$

Use this factor for your conversion as follows.

$$\frac{80 \cancel{\text{ mL}}}{1} \times \frac{1 \text{ L}}{1000 \cancel{\text{ mL}}} = 0.08 \text{ L}$$

Problems

Carry out the following conversions using the factor-label method.

1. How many seconds are in a year?
2. Convert 28 km to cm.
3. Convert 50 g to kg.
4. Convert 45 kg to mg.
5. Convert 450 m/s to m/h
6. Convert 50 liters to mL.
7. Convert 85 cm/min to m/s.
8. Convert the speed of light. $3.0 \times 10^8 \text{ m/s}$. to km/day